Worldline techniques for string theory solitons: recoil, annihilation and pair production.

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Abstract

We analyze a model of interacting particles and strings described by a path integral with the Dirichlet boundary conditions. Such model is a natural framework to examine the processes involving the center-of-mass motion of string theory D0-branes: recoil, annihilation and pair production. We demonstrate that, within the proposed formalism, the exclusive annihilation/pair-production amplitudes admit a saddle point evaluation. Even though the saddle point equation cannot be solved analytically, it allows to extract valuable information on the coupling constant dependence of the amplitudes. In particular, D0-brane pair production turns out to be suppressed as $\exp[-O(1/g_{st}^2)]$, much stronger than the naïve expectation $\exp[-O(1/g_{st})]$. All our derivations generalize rather immediately to the case of unstable D0-brane decay. In conclusion, we briefly comment on the possible implications our results may have for the conventional soliton-anti-soliton annihilation.

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Introduction

The physics of topological defects in relativistic field systems is a rather accomplished area of study [1], yet fairly little remains known about how such localized objects are pair-produced in collisions of elementary particles, and how they annihilate. It is of very little use that the scattering of the fundamental fields quanta off a topological defect (a process related to annihilation by crossing symmetry) has been analyzed in quite some detail. The masses of the solitons are typically inversely proportional to the coupling constant, forcing the annihilation products into the high energy regions of the phase space. Transition between the scattering and annihilation kinematic regions would then require an analytic continuation *non-perturbatively* far in the phase space, and the *perturbative* expansion of the amplitude in the scattering region will be essentially unrelated to the values of the amplitude in the annihilation region.

Even though there must exist a classical solution of the field equations describing the annihilation process, it does not appear to be accessible to any degree of analytic control. Indeed, there is apparently no straightforward way to prove that such a solution encapsulates even the most fundamental features to be expected from the annihilation process, e.g. the unitarity restrictions on the exclusive annihilation/pair-production amplitudes that we'll discuss below.

One may expect that the situation becomes even more involved for the solitonic objects of string theory, the D-branes, since, in that case, we would have to deal with all the complexities of quantum gravity in addition to the problems that plague the treatment of the field-theoretical process. It has been known since the seminal work by Polchinski [2] that the solitons of string theory admit a particularly simple description in terms of the Dirichlet boundary conditions for open strings. Indeed, the analysis of elementary quanta scattering in the presence of D-branes is substantially simpler than the corresponding problem in the conventional relativistic field theory, as it merely amounts to computing the expectation values of certain operators in a free 2-dimensional field system. One may therefore hope that worldsheet techniques would provide a valuable tool for understanding the D-brane annihilation, a tool unavailable to the corresponding field theory considerations.

Here, we attempt to construct a computational scheme for the D0-brane (D-particle) annihilation amplitude inspired by the simplicity of the CFT description of the extended D-branes. Since the center-of-mass motion of the D-particles is of a crucial importance for the processes we intend to consider, it is only natural that we introduce an explicit functional integration over the D-particle worldlines, to which the string worldsheets are attached via the Dirichlet boundary conditions. A similar method has been employed by Hirano and Kazama in their treatment of D-particle recoil [3]. It has been shown that, for low energy gravitational quanta, the scattering amplitudes obtained via this method can be matched with the standard no-recoil computation and respect the consistency requirements, such as BRST-invariance. Our goal in these notes is to address the question of how instructive this kind of approach can be for essentially high-energy processes, in particular, the D-particle annihilation. Similarly to Hirano and Kazama's paper, we restrict ourselves to bosonic D-particles, keeping in mind that the generalization to the potentially physically interesting case of superstring theory is likely to be conceptually straightforward, if only technically more challenging.

It is worth a notice that the attitude taken here is in many ways complementary to the attempts to address the same physical problem within the tachyon condensation paradigm of string field theory [4]. Indeed, the tachyon condensation approach typically deals with the initial state of coincident D-branes, whereas our aim is to analyze the genuine scattering[1]. On the other hand, should there be a need to generalize the approach advocated here to the case of higher dimensional D-branes, those will have to be wrapped around cycles in the compactification manifold, a requirement that does not appear within the string field theory considerations.

Speaking of the possible phenomenological applications, one should certainly mention the kinetics of primordial topological defects. Even though the inflationary scenario suggests that the density of primordial topological defects is dramatically diluted during the period of exponential expansion [6], they could play an important role at the earlier stages of the life of the Universe[2]. The problem we're considering is also in a (somewhat indirect) relation to the microscopic black hole pair production, a subject of much theoretical controversy in the recent past [8].

Determining the dependence of the annihilation amplitude on the coupling constant is the central theme of our present investigation. It is a requirement imposed by unitarity [9, 10] that the *exclusive* annihilation amplitudes for the topological defects must be non-perturbatively suppressed, i.e. the amplitude should vanish identically when expanded in powers of the coupling constant[3]. The general intuition about how such suppression could arise dynamically appeals to the well-known property that the inverse mass of the topological defects is typically much smaller (by powers of the coupling constant) than their size. Therefore, the typical wavelength of the annihilation products will be much smaller than the size of the objects they are produced by, resulting in exponential damping.

The problem with this kind of explanations is that the path integral includes arbitrarily singular configurations, and those could easily upset the exponential damping, which is supposed to underlie the non-perturbative suppression of the annihilation amplitude. (The precise meaning of this statement will become more apparent once we proceed with constructing the actual formalism.) The saddle point estimate of the amplitude we obtain in the following sections can be seen as a proof that the measure of those singular configurations in the path integral is too small to alter the coupling constant dependence necessary to maintain unitarity.

It is a fairly general principle that the processes involving topological defects upset the decreasing significance of the final states with large multiplicity familiar from the perturbative Feynman calculus. Producing a large number of quanta may often turn out to be advantageous compared to the low multiplicity final states. A possibly more familiar manifestation of this kind of behavior is the emission of a large number of gluons in the instanton-induced processes of gauge theories [11]. In our context, the bias for producing $O(1/g_{st})$ soft quanta

[1]A scattering-like process in a tachyon condensation setting has been recently considered in [5].

[2]In the cosmological context, the medium in which the topological defects move may exert substantial influence upon the annihilation process [7].

[3]Note that the *total* annihilation cross-section does not need to be non-perturbatively suppressed, and it is expected to be comparable to the square of the size of the topological defect for the case of gauge theory. This is one of the indications that the final state multiplicity distribution for topological defect annihilation is typically rather non-trivial.

is even more dramatic, as we'll be able to see after completing the saddle point evaluation of the amplitude.

Quantization of D-particle worldlines

When dealing with extended D-branes, one does not have to worry much about the center-of-mass motion: the infinite D-brane mass makes recoil impossible, at least when a localized object scatters off the extended D-brane. The situation is different, however, for compactified D-branes or genuine D-particles: for the low energy scattering experiments, the recoil is considerably suppressed by the mass of D-particle (inversely proportional to the coupling constant), but the center-of-mass motion becomes absolutely crucial in any essentially relativistic process, such as annihilation.

In the context of field theory, this problem has been addressed in [12]. One introduces the translational (center-of-mass) modes for the soliton explicitly, and performs quantization with the center-of-mass coordinate treated as a canonical variable. It has been a subject of some controversy how the center-of-mass degrees of freedom should be introduced for D0-branes [13, 14]. Hirano and Kazama's recipe [3] constitutes a specific proposal to this end.

To account for the motion of the D-particle as a whole, one introduces its coordinates explicitly and integrates over all the possible worldlines $f^\mu(t)$, with t being the proper time. The boundaries of the string worldsheet are restricted to the D-particle worldline, and the emission of closed strings is described by insertions of the closed string vertex operators in the interior of the worldsheet. Our main object of interest is the amplitude for two D-particles starting off at the positions x_1^μ and x_2^μ to annihilate into m closed strings carrying momenta k_1 to k_m:

$$\begin{aligned} G(x_1, x_2 | \, k_1, \cdots, k_m) = \sum \frac{(g_{st})^\chi}{V_{CK}} \int [\mathcal{D}f]_{\text{diff}} \mathcal{D}t \, \mathcal{D}X \, \delta\left(X_\mu(\theta) - f_\mu(t(\theta))\right) \\ \times \exp\left[-S_D(f) - S_{st}(X)\right] \prod_{a=1}^{m} \{g_{st} \mathcal{V}_a(k_a)\} \end{aligned} \tag{1}$$

where S_D is the action for the D-particle to be discussed below, S_{st} is the standard (conformal gauge) Polyakov action

$$S_{st} = \frac{1}{4\pi\alpha'} \int d^2\sigma \nabla X_\mu \nabla X^\mu,$$

the integration with respect to f_μ extends over all the inequivalent (unrelated by diffeomorphisms) curves starting at x_1 and ending at x_2, the boundary of the worldsheet is parametrized by θ, and $t(\theta)$ describes how this boundary is mapped onto the D-particle worldline. The sum is over all the topologies of the worldsheets (not necessarily connected, but without any disconnected vacuum parts) and χ is the Euler number. V_{CK} is the conformal Killing volume (the negative regularized value of [15] should be used for the disk). The fully integrated form of the vertex operators is implied. We work in the Euclidean space-time, keeping in mind a subsequent analytic continuation to the Minkowski signature. The integration over moduli of the worldsheet is suppressed for the rest of this paper, as it does not affect the qualitative results. The annihilation amplitude can be deduced from (1)

by means of the standard reduction formula:

$$\begin{aligned}
&\langle k_1, \cdots, k_m | p_1, p_2 \rangle \\
&\qquad = \lim_{p_1^2, p_2^2 \to -M^2} \left(p_1^2 + M^2\right)\left(p_2^2 + M^2\right) \int dx_1 dx_2 e^{ip_1 x_1} e^{ip_2 x_2} G(x_1, x_2 | \, k_1, \cdots, k_m)
\end{aligned} \tag{2}$$

where M is the D-particle mass.

Several general questions have to be addressed regarding the expression (1). Firstly, the issue with the Weyl invariance appears to be rather subtle. On the physical grounds, one would believe that making the D-branes fully dynamical reinforces the consistency of the amplitudes, much in the same way as respecting the supergravity equations of motion makes the non-linear σ-models consistent. We shall examine below how this works explicitly to the lowest order of the recoil perturbation theory. The case of annihilation/pair-production is considerably less computationally straightforward, and assessing the issues of the Weyl invariance there is likely to require some new ideas.

Of course, whether or not the integration over the D-particle worldlines reinforces the consistency of the string amplitudes depends crucially on the choice of the D-particle worldline action. It appears to be a fairly general principle [16] that the value of the effective action for a background which couples to strings is given by (minus) the sum of all connected vacuum string graphs evaluated in this background. Thus, very much in the spirit of [17]:

$$S_D[f] = \sum_{\text{connected}} \frac{(g_{st})^\chi}{-V_\chi} \int \mathcal{D}t \, \mathcal{D}X \, \delta\left(X_\mu(\theta) - f_\mu(t(\theta))\right) \exp\left[-S_{st}(X)\right] \tag{3}$$

Again, the negative regularized value of the conformal Killing volume should be used for the disk [15]. The exponentiation of the action in the path integral can be seen as a result of summing up the disconnected graphs containing vacuum parts [17]. It can be shown that, for nearly straight worldlines, the above action is reduced to the naïve point-particle result $M \int dt$. For curved worldlines, (3) would take into account the back-reaction from the space-time fields excited by the accelerating D-particle. Some properties of this action will become more apparent as we proceed with the computation of the amplitude.

Given the close relation between the integrand in (1) and the worldline action (3), it is convenient to rewrite (1) in the following form:

$$\begin{aligned}
G(x_1, x_2 | \, k_1, \cdots, k_m) = \sum_{\text{all}} \frac{\mathcal{C}\,(g_{st})^\chi}{V_{CK}} \int [\mathcal{D}f]_{\text{diff}} \mathcal{D}t \, \mathcal{D}X \, \delta\left(X_\mu(\theta) - f_\mu(t(\theta))\right) \\
\times \exp\left[-S_{st}(X)\right] \prod_{a=1}^{m} \left\{g_{st} \mathcal{V}_a(k_a)\right\}
\end{aligned} \tag{4}$$

Here, the sum extends over all the string diagrams including arbitrary disconnected vacuum parts. There is no explicit action for the D-particle worldline, but it arises after resumming the contributions from all the disconnected vacuum parts, which exponentiates to restore the S_D of (3). $\mathcal{C}$ is a combinatorial factor that can be deduced from (3).

Using the transformation properties of the vertex operators under the target space translations, it is easy to see that

$$G(x_1, x_2 | \, k_i) = \exp\left[\frac{i}{2}(x_1^\mu + x_2^\mu) \sum k_i\right] G\left(\frac{x_1 - x_2}{2}, -\frac{x_1 - x_2}{2} \middle| \, k_i\right)$$

The first term here merely provides for the momentum conservation δ-function in the Fourier transform, and (2) can be rewritten as

$$\begin{aligned}\langle k_1, \cdots, k_m | p_1, p_2 \rangle &= (2\pi)^{26} \delta(p_1 + p_2 + \sum k_i) \\ &\times \lim_{p_1^2, p_2^2 \to -M^2} \left(p_1^2 + M^2\right) \left(p_2^2 + M^2\right) \int dx \exp\left[\frac{i}{2}(p_1 - p_2)x\right] G\left(\frac{x}{2}, -\frac{x}{2} \middle| k_i\right)\end{aligned} \tag{5}$$

We shall work with this representation in our subsequent calculation of the amplitude.

The Gaussian integration

By a direct inspection of (4), it is easy to see that the integration over X is Gaussian and can be performed exactly. We will thus be able to recast the formalism into a (0+1)-dimensional form. The Gaussian integration we have to perform is closely related to the derivations in [16] and can be implemented by applying the formula:

$$\begin{aligned}&\int \mathcal{D}X \, \delta\left(X(\theta) - \xi(\theta)\right) \exp\left[\int d^2\sigma \left(-\frac{1}{4\pi\alpha'} \nabla X \nabla X + iJX\right)\right] \\ &= \exp\left[-\pi\alpha' \int J(\sigma) D(\sigma, \sigma') J(\sigma') d^2\sigma d^2\sigma' - i \int \xi(\theta) \partial_n D(\theta, \sigma') J(\sigma') d\theta d^2\sigma' \right. \\ &\qquad \left. - \frac{1}{4\pi\alpha'} \int \xi(\theta) \partial_n \partial_{n'} D(\theta, \theta') \xi(\theta') d\theta d\theta' \right]\end{aligned} \tag{6}$$

Here, D is the Dirichlet Green function of the Laplace operator $\Delta D(\sigma, \sigma') = -\delta(\sigma - \sigma')$, and ∂_n denotes the normal derivative evaluated at the boundary (which is parametrized by θ). It is convenient to consider

$$\begin{aligned}G\left(\frac{x}{2}, -\frac{x}{2} \middle| J\right) &= \sum_{\text{all}} \frac{\mathcal{C}\left(g_{st}\right)^\chi}{V_{CK}} \int [\mathcal{D}f]_{\text{diff}} \mathcal{D}t \, \mathcal{D}X \, \delta\left(X_\mu(\theta) - f_\mu(t(\theta))\right) \\ &\times \exp\left[-S_{st}(X) + i \int d^2\sigma J_\mu X^\mu\right]\end{aligned} \tag{7}$$

instead of (4). Indeed, differentiating with respect to the source J and setting it to $\sum k_i \delta(\sigma - \sigma_i)$ allows us to reproduce the amplitude for an arbitrary vertex operator insertion. Performing the integration in (7) by means of (6) yields:

$$\begin{aligned}G\left(\frac{x}{2}, -\frac{x}{2} \middle| J\right) &= \sum_{\text{all}} \frac{\mathcal{C}\left(g_{st}\right)^\chi}{V_{CK}} \exp\left[-\pi\alpha' \int J^\mu(\sigma) D(\sigma, \sigma') J_\mu(\sigma') d^2\sigma d^2\sigma'\right] \\ &\int [\mathcal{D}f]_{\text{diff}} \mathcal{D}t \, \exp\left[-i \int f^\mu(t(\theta)) \partial_n D(\theta, \sigma') J_\mu(\sigma') d\theta d^2\sigma'\right] \\ &\exp\left[-\frac{1}{4\pi\alpha'} \int f^\mu(t(\theta)) \partial_n \partial_{n'} D(\theta, \theta') f_\mu(t(\theta')) d\theta d\theta'\right]\end{aligned} \tag{8}$$

It should be noted that $D(\sigma, \sigma')$, χ, $\mathcal{C}$ and V depend on the topology of the diagram corresponding to each particular term in the sum. For diagrams with disconnected parts, $D(\sigma, \sigma')$

is block diagonal in the sense that it vanishes whenever the two arguments belong to different disconnected components.

The path integral in (8) may appear rather cumbersome, as one of the functions to be integrated over appears in the argument of the other one. Nevertheless, the integration over $f^\mu(t)$ can be performed exactly by means of a technique very similar to the treatment of the free point-like particle in [18].

We first rewrite the measure on reparametrization equivalence classes of $f^\mu(t)$ as

$$[\mathcal{D}f]_{\text{diff}} = \mathcal{D}f\,\delta\left[\dot{f}^2 - 1\right] = \mathcal{D}f \int \mathcal{D}z \exp\left[-\int_0^T z(\dot{f}^2 - 1)dt\right]$$

where $\delta[\dot{f}^2 - 1]$ is a product of δ-functions at every point (reinforcing t to be the proper time), and, for each t, the integration over $z(t)$ is along a contour going from $c - i\infty$ to $c + i\infty$ in the complex z plane, with c being an arbitrary (positive) constant. This contour can of course be deformed, an opportunity implicit in our subsequent application of the saddle point method.

If we now introduce

$$\mathcal{N}(t,t') = \int d\theta d\theta' \partial_n \partial_{n'} D(\theta,\theta')\,\delta\left(t - t(\theta)\right)\delta\left(t' - t(\theta')\right), \quad d(t,\sigma) = \int d\theta \partial_n D(\theta,\sigma)\,\delta\left(t - t(\theta)\right)$$

the f-integration in (8) can be recast into a manifestly Gaussian form:

$$G\left(\frac{x}{2}, -\frac{x}{2}\middle| J\right) = \sum_{\text{all}} \frac{\mathcal{C}\,(g_{st})^\chi}{V_{CK}} \exp\left[-\pi\alpha' \int J^\mu(\sigma) D(\sigma,\sigma') J_\mu(\sigma') d^2\sigma d^2\sigma'\right]$$
$$\int \mathcal{D}t\,\mathcal{D}z\,\mathcal{D}f \exp\left[-\int z(\dot{f}^2 - 1)dt\right] \exp\left[-i\int f^\mu(t) d(t,\sigma) J_\mu(\sigma) dt d^2\sigma'\right]$$
$$\exp\left[-\frac{1}{4\pi\alpha'}\int f^\mu(t)\mathcal{N}(t,t') f_\mu(t') dt dt'\right]$$

It is convenient to pass on to the integration over $\mathcal{D}\dot{f}$ by means of the relations

$$\mathcal{D}f = dT\mathcal{D}\dot{f}\,\delta\left(\int_0^T \dot{f}^\mu dt + x^\mu\right) \qquad f^\mu(t) = \frac{x}{2} + \int_0^t \dot{f}^\mu dt$$

We should also keep in mind that

$$\int_0^T dt \mathcal{N}(t,t') = 0 \qquad \int_0^T dt\, d(t,\sigma) = -1$$

as

$$\int d\theta\,\partial_n\partial_{n'} D(\theta,\theta') = 0 \qquad \int d\theta\,\partial_n D(\theta,\sigma') = -1$$

If we also perform the Fourier transform of (5), we arrive at the following representation

$$
\begin{aligned}
G(p_1,p_2|J) &= \int dx \exp\left[\frac{i}{2}(p_1-p_2)x\right] G\left(\frac{x}{2},-\frac{x}{2}\bigg|\, J\right) \\
&= \sum_{\text{all}} \frac{\mathcal{C}\,(g_{st})^{\chi}}{V_{CK}} \exp\left[-\pi\alpha' \int J^{\mu}(\sigma)D(\sigma,\sigma')J_{\mu}(\sigma')d^2\sigma d^2\sigma'\right] \\
&\qquad \int dT\mathcal{D}t\,\mathcal{D}z\; e^{\int z\,dt} \int \mathcal{D}\dot{f}\; \exp\left[i\int \dot{f}_{\mu}(t)\left(p_1^{\mu} - \int_0^t d(\tilde{t},\sigma)J^{\mu}(\sigma)d\tilde{t}d^2\sigma\right)dt\right] \\
&\qquad \exp\left[-\int \dot{f}^{\mu}(t)\mathcal{B}(t,t')\dot{f}_{\mu}(t')dtdt'\right]
\end{aligned}
$$

where we've introduced

$$
\mathcal{B}(t,t') = z(t)\,\delta(t-t') + \frac{1}{4\pi\alpha'}\int_0^t d\tilde{t}\int_0^{t'} d\tilde{t}'\,\mathcal{N}(\tilde{t},\tilde{t}')
$$

At this point, the Gaussian integration becomes completely straightforward and yields

$$
\begin{aligned}
G(p_1,p_2|J) = \sum_{\text{all}} &\frac{\mathcal{C}\,(g_{st})^{\chi}}{V_{CK}} \exp\left[-\pi\alpha' \int J^{\mu}(\sigma)D(\sigma,\sigma')J_{\mu}(\sigma')d^2\sigma d^2\sigma'\right] \\
&\int dT\mathcal{D}t(\theta)\,\mathcal{D}z(t)\;\det[\mathcal{B}]^{-1/2}\exp\left[\int z\,dt\right] \\
\exp\Bigg[-\frac{1}{4}\int &\Big(p_1^{\mu} - \int_0^t d(\tilde{t},\sigma)J^{\mu}(\sigma)d\tilde{t}d^2\sigma\Big)\mathcal{B}^{-1}(t,t')\Big(p_{1\mu} - \int_0^{t'} d(\tilde{t}',\sigma')J_{\mu}(\sigma')d\tilde{t}'d^2\sigma'\Big)dtdt'\Bigg]
\end{aligned}
\tag{9}
$$

From this representation, it is apparent that the endpoints of the z integration contour can be moved towards $-\infty$. The contour itself cannot shrink to $-\infty$, however, on account of the singularities of $(\det\mathcal{B})^{-1/2}$. Should there be a discontinuity in $t(\theta)$, these singularities move towards $-\infty$ allowing the contour to be deformed arbitrarily far to the left in the complex z plane. Then the integrand will vanish due to the last factor in the second line of (9). This is as it should be, since discontinuous worldsheets do not give any contribution to the original path integral (1).

The fact that we have been able to perform the integration over the D-particle worldlines exactly is rather remarkable, since, for any *given* worldline, the action S_D featured in (1) cannot be computed. This action however is itself constructed from string amplitudes and can be completely eliminated from the path integral at the cost of including the disconnected string worldsheets, as it has been done in (4). This peculiar simplification plays an important role in making possible the saddle point evaluation of the annihilation/pair-production amplitudes.

The saddle point method

The presence of the p_1 and J in the exponential of the last line of (9) alludes to the relevance of the saddle point techniques, as those quantities become non-perturbatively large in the

annihilation kinematic region. This can be made more apparent by introducing $\wp_i = p_i/M$ and $\mathcal{J} = J/M$ (M being the D-particle mass) and rewriting (9) as

$$G(p_1, p_2|J) = \sum_{\text{all}} \frac{\mathcal{C}\,(g_{st})^\chi}{V_{CK}} \exp\left[-\pi\alpha' M^2 \int \mathcal{J}^\mu(\sigma) D(\sigma,\sigma') \mathcal{J}_\mu(\sigma') d^2\sigma d^2\sigma'\right]$$
$$\int dT \mathcal{D}t(\theta)\, \mathcal{D}z(t)\, \det[\mathcal{B}]^{-1/2} \exp\left[\int z\, dt\right]$$
$$\exp\left[-\frac{M^2}{4}\int \Big(\wp_1^\mu - \int\limits_0^t d(\tilde{t},\sigma)\mathcal{J}^\mu(\sigma) d\tilde{t} d^2\sigma\Big)\mathcal{B}^{-1}(t,t')\Big(\wp_{1\mu} - \int\limits_0^{t'} d(\tilde{t}',\sigma')\mathcal{J}_\mu(\sigma') d\tilde{t}' d^2\sigma'\Big) dt dt'\right]$$

The latter representation is still somewhat inconvenient, since the position of the saddle point depends on the saddle point parameter M. This dependence is very simple, however, and can be completely eliminated by rescaling[4] $z(t) \to Mz(t/4\pi M\alpha')$, $t(\theta) \to 4\pi M\alpha' t(\theta)$, $T \to 4\pi M\alpha' T$, which brings the above integral to the form

$$G(p_1, p_2|J) = \sum_{\text{all}} \frac{\mathcal{C}\,(g_{st})^\chi}{V_{CK}} \exp\left[-\pi\alpha' M^2 \int \mathcal{J}^\mu(\sigma) D(\sigma,\sigma') \mathcal{J}_\mu(\sigma') d^2\sigma d^2\sigma'\right]$$
$$\int dT \mathcal{D}t(\theta)\, \mathcal{D}z(t)\, \det[\mathcal{B}]^{-1/2} \exp\left[4\pi\alpha' M^2 \int z\, dt\right] \tag{10}$$
$$\exp\left[-\pi\alpha' M^2 \int \Big(\wp_1^\mu - \int\limits_0^t d\cdot \mathcal{J}^\mu d\tilde{t} d^2\sigma\Big)\mathcal{A}^{-1}(t,t')\Big(\wp_{1\mu} - \int\limits_0^{t'} d\cdot \mathcal{J}_\mu d\tilde{t}' d^2\sigma'\Big) dt dt'\right]$$

where

$$\mathcal{A}(t,t') = z(t)\,\delta(t-t') + \int\limits_0^t d\tilde{t} \int\limits_0^{t'} d\tilde{t}'\, \mathcal{N}(\tilde{t},\tilde{t}')$$

In view of the subsequent application of the saddle point method, it is necessary to specify how the integrand is analytically continued to the complex $t(\theta)$. It may seem worrisome that the definitions of $\mathcal{N}(t,t')$ and $d(t,\sigma)$ involve δ-functions, but these δ-functions appear in integral convolution, and the result can be made manifestly analytic by expressing them through their Fourier series on the interval $[0,T]$. Such a representation defines $\mathcal{N}(t,t')$ and $d(t,\sigma)$ as periodic with respect to shifting t or t' by multiples of T. It is worth a notice that analogous analytic continuation is not possible at the level of (8) due to the lack of smoothness[5] in $f^\mu(t)$. In this respect, the summation over all the worldlines performed in the previous section is a crucial prerequisite for the saddle point techniques to be applicable.

With these specifications, the saddle point equations can be readily derived by requiring the sum of the two exponents in (10) to be stationary with respect to variations of $t(\theta)$ and

[4] It should be kept in mind that, from now on, the geometrical parameters of the worldsheet in physical space are related to those represented by $t(\theta)$ through a factor of $O(1/g_{st})$.

[5] It is a familiar fact (related to the central limit theorem) that the measure on random paths is dominated by curves of fractal dimension 2: see, for example, [19].

$z(t)$. In this manner, one obtains

$$\mathcal{P}^\mu(t(\theta))\left(\int d\theta'\,\partial_n\partial_{n'}D(\theta,\theta')\int\limits_0^{t(\theta')}\mathcal{P}_\mu(t')dt' + \int\partial_n D(\theta,\sigma)\mathcal{J}_\mu(\sigma)d^2\sigma\right) = 0 \tag{11}$$

$$\mathcal{P}^\mu\mathcal{P}_\mu = -4 \tag{12}$$

where we've introduced

$$\mathcal{P}^\mu(t) = \int\limits_0^T dt'\mathcal{A}^{-1}(t,t')\left(\wp_1^\mu - \int\limits_0^{t'} d(\tilde{t}',\sigma')\mathcal{J}^\mu(\sigma')d\tilde{t}'d^2\sigma'\right)$$

Multiplying (11) by $\delta(t-t(\theta))$ and integrating with respect to θ, we get

$$\dot{z}\,\mathcal{P}^2 + \frac{1}{2}z\left(\mathcal{P}^2\right)^{\cdot} = 0$$

which, coupled with (12), implies $z = \text{const}$. The relevant value of the constant can be determined from the following argument. Because of the subsequent application of the reduction formula, the value of the amplitude is determined by the divergent piece of $G(p_1,p_2|J)$ as the momenta go on-shell. This divergence can only come from large values of T in the integral (which, of course, corresponds to the long distance propagation of the on-shell particles). For large values of T, there will always be parts of the worldline arbitrarily remote from the location of the worldsheet. There, (12) will fix $z = \pm i\sqrt{\wp^2}/2$ (with $\wp^2 = \wp_1^2 = \wp_2^2$). Since the saddle point value of z does not depend on t, we conclude that, for the purposes of evaluating the amplitude, z can be set to $\pm i\sqrt{\wp^2}/2$ everywhere.

Thus, we have two complex conjugate saddle points for z. After substituting the corresponding values to (11), we should obtain two complex conjugate solutions for $t(\theta)$. The equation (11) is clearly beyond the reach of analytic methods, but the saddle point configuration of the disconnected vacuum components of the worldsheet can be easily identified as $t(\theta) = \text{const}$. Indeed, for θ belonging to a disconnected vacuum component, the second term in the brackets of (11) vanishes. Enforcing $t(\theta) = \text{const}$ makes the first term in the brackets vanish as well, the equation being thereby satisfied.

Moreover, the values of the second functional derivative of the saddle point functional with respect to $t(\theta)$ can be calculated for the parts of the worldsheet which do not emit any final state particles. Indeed, if θ and θ' both belong to a disconnected vacuum component with $t(\theta) = t_0$,

$$\frac{\delta^2}{\delta t(\theta)\delta t(\theta')}\left[\int\Big(\wp_1^\mu + \int\limits_0^t d\cdot\mathcal{J}^\mu d\tilde{t}d^2\sigma\Big)\mathcal{A}^{-1}(t,t')\Big(\wp_{1\mu} + \int\limits_0^{t'} d\cdot\mathcal{J}_\mu d\tilde{t}'d^2\sigma'\Big)\right]$$

$$= -2\mathcal{P}^2(t_0)\partial_n\partial_{n'}D(\theta,\theta') = 8\,\partial_n\partial_{n'}D(\theta,\theta')$$

Furthermore, if θ belongs to a disconnected vacuum component and θ' belongs to any other component of the worldsheet, the corresponding second derivative vanishes.

The above specifications in fact allow to perform a complete resummation (to the leading order of the saddle point approximation) of the contributions to (10) coming from the disconnected vacuum components. Indeed, for each disconnected component in a given diagram, we'll obtain a factor

$$\int \mathcal{D}t(\theta)\exp\left[-8\pi\alpha' M^2\int t(\theta)\partial_n\partial_{n'}D(\theta,\theta')t(\theta')d\theta d\theta'\right] \tag{13}$$

The value of this integral is a constant[6] (depending on the topology of each particular disconnected vacuum component) times a factor of T, which comes from the integration over the constant mode of $t(\theta)$. If we now recall that the combinatorial coefficients $\mathcal{C}$ in (10) originated from expanding the exponential in (1), it is easy to see that, to the leading order of the saddle point approximation, the effect of all the disconnected vacuum components amounts to a factor of $\exp[\mu T]$, where μ is a constant, which can be evaluated by computing the Gaussian integral (13). In fact, it is easier to notice that $G(p_1,p_2|J=0)$ is merely the D-particle propagator, and, identifying its pole with the D-particle mass M, conclude that $\mu=-4\pi\alpha' M^2$.

For the parts of the worldsheet which do emit final state strings, it is not possible to solve the equation (11) explicitly. However, valuable information can be extracted from the mere assumption that a saddle point exists (this can be checked in the low energy scattering case). It is an important circumstance that, for the worldsheets located not too close to the endpoints of the worldline, in the limit $T\to\infty$, the value of the saddle point function does not change under the shifts of $t(\theta)$ by a constant (we assume $\wp_1^2=\wp_2^2=\wp^2$, as it is on-shell). It is not hard to take into account this quasi-zero mode though, as the subsequent application of the reduction formula discards everything but the term growing most rapidly as T goes to ∞. Since extending the interval to which t and t' belong to $[-\infty,\infty]$ does not introduce any singularities to (11), we should expect that, as T goes to ∞, the solutions to (11) located not too close to the endpoints of the worldline approach a fixed shape. The saddle point evaluation of the $\mathcal{D}t$ and $\mathcal{D}z$ integrals in (10) can be therefore written (up to the pre-exponential factors) as

$$e^{-4i\pi\alpha' M^2 T\sqrt{\wp^2}}\left(Te^{-\alpha' M^2\mathcal{F}[\mathcal{J}]}+o(T)\right) \tag{14}$$

where $\mathcal{F}$ does not depend on T and can be determined from the solution to the equation (11) with t and t' running from $-\infty$ to ∞. The above expression is written for the saddle point $z=i\sqrt{\wp^2}/2$, the contribution from $z=-i\sqrt{\wp^2}/2$ is the complex conjugate of this. We shall omit the $o(T)$ term in the following, as it vanishes upon the application of the reduction formula.

Assembling everything together, up to the pre-exponential factors, the saddle point esti-

[6] One may argue that, for higher genus worldsheets, the modular integrations (that we do not write explicitly) will exhibit a tachyonic divergence. The same problem arises if one tries to compute the higher genus corrections to the D-particle mass. Both problems will disappear in the superstring case, which we see as a conceptual rationale behind the formal identification of the value of the integral (13) with the D-particle mass, which we're about to make.

mate of (10) is

$$\Big[G(p_1,p_2|J)\Big]_{\text{saddle}} \approx \sum \frac{(g_{st})^{\chi}}{V_{CK}} \exp\left[-\pi\alpha' M^2 \int \mathcal{J}^{\mu}(\sigma) D(\sigma,\sigma') \mathcal{J}_{\mu}(\sigma') d^2\sigma d^2\sigma'\right]$$
$$\times\, e^{-\alpha' M^2 \mathcal{F}[\mathcal{J}]} \int_0^{\infty} T \exp\left[-4\pi\alpha' M^2 \left(1 + i\sqrt{\wp^2}\right) T\right] dT + \text{c.c.}$$

where the summation is once again performed only over those worldsheets which do not contain any disconnected vacuum parts. Performing the T integration and applying the reduction formula, we finally obtain

$$\begin{aligned}\langle p_1,p_2|J\rangle_{\text{saddle}} \approx \sum \frac{(g_{st})^{\chi}}{V_{CK}} \exp\left[-\pi\alpha' \int J^{\mu}(\sigma) D(\sigma,\sigma') J_{\mu}(\sigma') d^2\sigma d^2\sigma'\right] \\ \times \left(\exp\left\{-\alpha' M^2 \mathcal{F}[J/M]\right\} + \text{c.c.}\right)\end{aligned} \tag{15}$$

To compute the actual amplitude, one has to combine this expression and its functional derivatives with respect to J so as to imitate the vertex operator insertions, as well as to integrate over the worldsheet moduli and the positions of the vertex operators (these will be saddle point integrations themselves, in analogy to [20]). All these tasks are likely to require numerical computations, as is the evaluation of the $\mathcal{F}$-functional.

An essential piece of information contained in the expression (15) is, however, that it identifies the dependence of the annihilation amplitude on the coupling constant. We can see that, for a final state with the number of quanta fixed as $g_{st} \to 0$ and momenta growing proportionally to the mass of the D-particle, the amplitude diminishes as $\exp(-O(1/g_{st}^2))$. Indeed, the anticipated non-perturbative suppression has become manifest[7]. It is worth noting that the character of this coupling dependence is not at all obvious at the intermediate stages of our computation. Only upon the application of the saddle point method can we see that the qualitative arguments outlined in the introduction do, in fact, result in a non-perturbative suppression of the amplitude.

It may appear rather surprising that the estimate of the amplitude obtained above is exactly the same as if we had used the simplistic free particle action $M\int dt$ in place of the expression (3) which takes into account the emission and re-absorption of virtual string states by the accelerating D-particles. The qualitative explanation here is that, being objects of effective size $\sqrt{\alpha'}$, the D-particles are not likely to emit *virtual* string states of energy much higher than $1/\sqrt{\alpha'}$. These states will produce worldline curvatures of order g_{st} and will not therefore affect the leading order of the saddle point approximation. One may have suspected that the worldline curvature could become large in the region where the *final* state strings are emitted. Yet, the saddle point equation (11) implies that the amplitude is dominated by worldsheets of physical size $\sim 1/g_{st}$ (which corresponds to $t(\theta)$ being of order 1). Therefore, the emission occurs from a segment of the worldline whose size is of order $1/g_{st}$, and the relevant curvatures are of order g_{st} again.

[7] There doesn't appear to be any straightforward way to prove that $\mathcal{F}$ is greater than zero in the entire annihilation kinematic region (after the analytic continuation to Minkowski signature). Nevertheless, (15) suggests that the amplitude is either non-perturbatively suppressed or non-perturbatively enhanced, and the inherent absurdity of the latter option is quite apparent.

Recoil perturbation theory and the Fischler-Susskind mechanism

Let us now take a step back and examine how the more conventional coupling expansion of the recoil perturbation theory arises in our set-up. Namely, let's set aside the (necessarily relativistic) annihilation/pair-production processes and consider closed string states scattering off a D-particle. Since the mass of the D-particle diverges in the limit $g_{st} \to 0$, it can be treated as static to the lowest order in g_{st} (if we keep the momenta of the incident closed strings fixed as $g_{st} \to 0$), and the corrections due to the motion of the D-particle's center-of-mass (i.e. recoil) will appear as a perturbative expansion in powers of g_{st}. This is the familiar recoil perturbation theory.

The formalism of the previous section is, of course, perfectly applicable in this case. Since the momenta of the closed string states (and hence J) are kept fixed as $g_{st} \to 0$ (and hence $M \to \infty$), $\mathcal{J} \equiv J/M$ can be treated as perturbation. The saddle point equation (11) implies then that the saddle point configuration $t(\theta) = \text{const} + O(1/M)$, i.e. the path integral (10) is dominated by nearly constant $t(\theta)$ (the fluctuations of $t(\theta)$ are of order $1/M$ generically, and so is the saddle point value, as we've just remarked). One could look for solutions of the saddle point equation as an expansion in $\mathcal{J}$. However, it is much more efficient to generate the perturbation theory directly from the path integral (10) by constructing a suitable expansion of the "effective action" functional of (10)

$$S_{eff}[z(t), t(\theta)] = -4\pi\alpha' M^2 \int z\, dt$$
$$+\pi\alpha' M^2 \int \Big(\wp_1^\mu - \int\limits_0^t d\cdot \mathcal{J}^\mu d\tilde{t} d^2\sigma\Big) \mathcal{A}^{-1}(t,t') \Big(\wp_{1\mu} - \int\limits_0^{t'} d\cdot \mathcal{J}_\mu d\tilde{t}' d^2\sigma'\Big) dt dt'$$

There is one difficulty one encounters in implementing such a program. It is a most straightforward approach to try to construct a Taylor-like expansion of S_{eff} in powers of $t(\theta)$ around $t(\theta) = \text{const}$:

$$S_{eff}(\text{const} + t(\theta))$$
$$= S_{eff}(\text{const}) + \int d\theta \left.\frac{\delta S_{eff}}{\delta t(\theta)}\right|_{\text{const}} t(\theta) + \frac{1}{2} \int d\theta d\theta' \left.\frac{\delta^2 S_{eff}}{\delta t(\theta) \delta t(\theta')}\right|_{\text{const}} t(\theta) t(\theta') + \cdots$$

This strategy comes to mind in immediate relation to the computational techniques most commonly used in σ-models, and it has been employed in [3] for the purposes we're presently pursuing here. Unfortunately, such an expansion does not exist. In [3], various ζ-function prescriptions have been devised (in the lowest order) to deal with the infinities arising when one tries to brute-force the Taylor-like expansion, but the situation quickly becomes hopeless, if one tries to envisage the general structure of the recoil perturbation theory in such a framework.

The origin of the above complication can be traced back to the non-analytic properties of the worldlines in the path integral (8). Indeed, for non-analytic $f^\mu(t)$ (most worldlines are fractal and therefore non-differentiable [19]) the "effective action" in (8) does not admit a Taylor-like expansion in $t(\theta)$ around *any* configuration of $t(\theta)$. The integration over $f^\mu(t)$

improves the situation considerably: the resulting effective action can be expanded around any $t(\theta) \neq$ const, but the non-analyticity still survives for the worldsheets whose boundary shrinks to a single point. (One may become worried about whether such non-analyticity could undermine the application of the saddle point method from the previous section. Whereas any discussions of explicit contour deformation in multi-dimensional case are necessarily rather subtle, there are still good chances that the deformation implicit in the saddle point evaluation of the integral can be attained, since S_{eff} *is* analytic for any $t(\theta) \neq$ const.)

Luckily, the Taylor-like expansion is not the only way to generate a sensible perturbation theory. Appearing as insertions in a Gaussian path integral, the exponentials of $t(\theta)$ are just as tractable as powers of $t(\theta)$. We shall therefore resort to a combination of a Taylor-like and a Fourier-like expansion. We first introduce

$$A(t,t') = i\frac{\sqrt{\wp^2}}{2}\delta(t-t') \qquad B(t,t') = \delta z(t)\,\delta(t-t') + \int\limits_0^t d\tilde{t}\int\limits_0^{t'} d\tilde{t}'\,\mathcal{N}(\tilde{t},\tilde{t}')$$

such that $\mathcal{A}(t,t') = A(t,t') + B(t,t')$, and expand formally

$$\mathcal{A}^{-1} = \frac{1}{A} - \frac{1}{A}B\frac{1}{A} + \frac{1}{A}B\frac{1}{A}B\frac{1}{A} + \cdots \tag{16}$$

(we work with one of the two saddle points in $z(t)$, the other one will give a complex conjugate contribution, as in the previous section). Upon inserting this expression in the S_{eff} we can isolate the following terms that need to be retained in the exponent (everything else is small and may be treated perturbatively, as it will become apparent after we exhibit the powers of the expansion parameter $1/M$):

$$\begin{gathered}
S^{(1)}_{eff} = 4\pi i\alpha' M^2\sqrt{\wp^2}T \\
S^{(2)}_{eff} = -4\pi\alpha' M^2\int dtdt'\int\limits_0^t d\tilde{t}\int\limits_0^{t'} d\tilde{t}'\,\mathcal{N}(\tilde{t},\tilde{t}') \equiv -4\pi\alpha' M^2\int t(\theta)\partial_n\partial_{n'}D(\theta,\theta')t(\theta')d\theta d\theta' \\
S^{(3)}_{eff} = -\frac{8\pi i\alpha' M^2}{\sqrt{\wp^2}}\int \delta z^2 dt \\
S^{(4)}_{eff} = \frac{4\pi i\alpha' M^2}{\sqrt{\wp^2}}\wp_1^\mu\int dt\int\limits_0^t d\cdot\mathcal{J}_\mu d\tilde{t}d^2\sigma \equiv \frac{4\pi i\alpha' M}{\sqrt{\wp^2}}\wp_1^\mu\int t(\theta)\partial_n D(\theta,\sigma)J(\sigma)_\mu d\theta d^2\sigma
\end{gathered} \tag{17}$$

The first term here merely provides for the correct pole structure. It will disappear after integration over T and application of the reduction formula. The remaining three terms define a Gaussian integral with respect to $z(t)$ and $t(\theta)$.

A general term in the expansion (16) can be constructed as follows. We should take a certain number of factors

$$\int\limits_0^{t_i} d\tilde{t}\int\limits_0^{t_{i+1}} d\tilde{t}'\,\mathcal{N}(\tilde{t},\tilde{t}') \tag{18}$$

and, for each i add a factor

$$\frac{(\delta z(t_i))^{n_i}}{(i\sqrt{\wp^2}/2)^{-n_i+1}}$$

where n_i are non-negative integers. Then, for the first and last t_i we either add a factor of $\wp_1^\mu$ or a factor of

$$\int_0^{t_i} d(\tilde{t},\sigma)\mathcal{J}^\mu(\sigma)d\tilde{t}d^2\sigma \tag{19}$$

We then integrate over all t_i's and multiply the result by $\pi\alpha' M^2$.

We now proceed with the Gaussian integral over $\delta z(t)$. As it can be seen from the expression for $S_{eff}^{(3)}$, each factor of $\delta z(t)$ will produce a factor of $1/M$ in the result of the Gaussian integration. Where several δz's occur at the same t_i, divergencies from the singularity of the δz propagator (proportional to the δ-function) will be present. Just like in the case of free particle considered in [18], these divergencies will merely renormalize the saddle point (expectation) value of $z(t)$.

We are left with a Gaussian integral over $t(\theta)$. As we've remarked before, the structures of the type (18) and (19) cannot be expanded in powers of $t(\theta)$ around $t(\theta) = \text{const}$. Instead, we'll Fourier-trasform in t_i's:

$$\int_0^{t_i} d\tilde{t} \int_0^{t_{i+1}} d\tilde{t}' \mathcal{N}(\tilde{t},\tilde{t}') \to \frac{1}{\kappa_i \kappa_{i+1}} \int e^{i\kappa_i t(\theta)} \partial_n \partial_n' D(\theta,\theta') e^{i\kappa_{i+1} t(\theta')} d\theta d\theta'$$

$$\int_0^{t_i} d(t,\sigma)\mathcal{J}^\mu(\sigma)d\tilde{t}d^2\sigma \to \frac{1}{\kappa_i} \int e^{i\kappa_i t(\theta)} \partial_n D(\theta,\sigma)\mathcal{J}^\mu(\sigma) d\theta d^2\sigma$$

At this point, the integration over $t(\theta)$ can be explicitly performed. The substitution $t(\theta) \to t(\theta)/M$, $\kappa_i \to M\kappa_i$ reveals the additional powers of $1/M$ that each term will receive upon the integration over $t(\theta)$.

Having sketched the general structure of the recoil perturbation theory, we shall now turn to the important issue of how the consistency of the string amplitudes is respected to the lowest non-trivial order. As we've remarked early on in the course of our present investigation, the inclusion of curved D-particle worldlines into the path integral (1) will generally induce a Weyl anomaly thereby threatening the decoupling of the negative norm states, which is a crucial requirement for the consistency of the string theory S-matrix. The educated hope is that (a suitable modification of) the familiar Fischler-Susskind mechanism [21, 22] will come to rescue the formalism that has been constructed so far. Indeed, it is a well known fact that there are singularities in the modular integration for the higher genus worldsheets coming from the corners of the moduli space. Should we have chosen the D-particle worldline action correctly, the Weyl anomalies induced by those singularities will precisely cancel the Weyl anomalies coming from the coupling to an accelerating D-particle. Let us see how this works out to the first non-trivial order of the recoil perturbation theory.

The Fischler-Susskind mechanism in our set-up implies a cancellation between an infrared divergence on a higher genus worldsheet and an ultraviolet divergence on a lower genus worldsheet (in the sense of cancellation of the corresponding Weyl anomalies). To the lowest order, this means that we should examine the ultraviolet divergencies on a disk to the order $1/M$ and compare them with the modular integration divergencies from an annulus coupled to a straight D-particle worldline (the curved worldlines will only contribute to the higher orders in $1/M$).

When we expand S_{eff} (for a disk) according to (16) and treat all the terms except for the ones indicated in (17) as a perturbation, we can identify the following insertions that can in principle contribute to the order $1/M$ in the resulting recoil perturbation theory Gaussian integral:

$$F_1 = -\frac{2\pi i\alpha' M^2}{\sqrt{\wp^2}}\int dt \left(\int_0^t d(\tilde{t},\sigma)\mathcal{J}^\mu(\sigma) d\tilde{t} d^2\sigma\right)^2$$

$$F_2 = \frac{8\pi\alpha' M^2}{\wp^2}\wp^\mu \int dt dt' \int_0^t d\tilde{t}\int_0^{t'} d\tilde{t}' \,\mathcal{N}(\tilde{t},\tilde{t}')\int_0^{t'} d\tilde{t}'' d^2\sigma d(\tilde{t}'',\sigma)\mathcal{J}^\mu(\sigma)$$

$$F_3 = \frac{1}{2}\left(\frac{8\pi\alpha' M^2}{\wp^2}\wp^\mu \int dt \delta z(t)\int_0^t d(\tilde{t},\sigma)\mathcal{J}^\mu(\sigma) d\tilde{t} d^2\sigma\right)^2$$

After we integrate over $\delta z(t)$ and Fourier-transform, these three terms become:

$$F_1 \sim \alpha' \int \frac{d\kappa}{\kappa^2}\left|\int e^{i\kappa t(\theta)}\partial_n D(\theta,\sigma) J_\mu(\sigma) d\theta d^2\sigma\right|^2$$

$$F_2 \sim \alpha' M \wp_1^\mu \int \frac{d\kappa}{\kappa^2}\int d\theta d\theta' t(\theta)\partial_n\partial_n' D(\theta,\theta') e^{i\kappa t(\theta')}\int e^{-i\kappa t(\theta'')}\partial_n D(\theta'',\sigma) J_\mu(\sigma) d\theta'' d^2\sigma$$

$$F_3 \sim \alpha' \int \frac{d\kappa}{\kappa^2}\left|\wp_1^\mu \int e^{i\kappa t(\theta)}\partial_n D(\theta,\sigma) J_\mu(\sigma) d\theta d^2\sigma\right|^2$$

We now have to compute the path integral

$$\int \mathcal{D}t(\theta)\ (F_1+F_2+F_3)\exp\left[-S^{(2)}_{eff} - S^{(4)}_{eff}\right]$$

The relevant structure that is featured in all the three terms is

$$\begin{aligned}
&\int \frac{d\kappa}{\kappa^2}\int \mathcal{D}t(\tilde{\theta})\, e^{i\kappa(t(\theta)-t(\theta'))} \\
&\times \exp\left[-4\pi\alpha' M^2 \int t(\tilde{\theta})\partial_n\partial_{n'} D(\tilde{\theta},\tilde{\theta}') t(\tilde{\theta}') d\tilde{\theta} d\tilde{\theta}' + \frac{4\pi i\alpha' M}{\sqrt{\wp^2}}\wp_1^\mu \int t(\tilde{\theta})\partial_n D(\tilde{\theta},\sigma) J_\mu(\sigma) d\tilde{\theta} d^2\sigma\right] \\
&\quad \sim \frac{1}{\alpha' M}\exp\left[-\pi\alpha' \wp_1^\mu \wp_1^\nu \int J_\mu(\sigma)\partial_n D(\tilde{\theta},\sigma)[\partial_n\partial_{n'} D]^{-1}(\tilde{\theta},\tilde{\theta}')\partial_{n'} D(\tilde{\theta}',\sigma') J_\nu(\sigma') d\tilde{\theta} d\tilde{\theta}' d^2\sigma d^2\sigma'\right] \\
&\qquad\qquad \times \int \frac{d\kappa}{\kappa^2}\exp\left[-2\kappa^2\log\varepsilon + \kappa h_1(\theta,\theta') + \kappa^2 h_2(\theta,\theta')\right]
\end{aligned}$$

Where $\log\varepsilon \equiv \partial_n\partial_{n'} D(\theta,\theta)$ is the regularized value of the singularity of the boundary-to-boundary propagator (ε being the worldsheet cut-off), and $h_{1,2}(\theta,\theta')$ are certain (cut-off independent) functions. If we now substitute $\tilde{\kappa} = \kappa/\sqrt{\log\varepsilon}$, we observe that the functions $h_{1,2}$ do not contribute to the UV divergent piece of the path integral[8], which turns out to be (θ,θ')-independent and proportional to

$$\frac{\sqrt{\log\varepsilon}}{\alpha' M} e^{-\pi\alpha' \wp_1^\mu \wp_1^\nu \int J_\mu(\sigma)\partial_n D(\tilde{\theta},\sigma)[\partial_n\partial_{n'} D]^{-1}(\tilde{\theta},\tilde{\theta}')\partial_{n'} D(\tilde{\theta}',\sigma') J_\nu(\sigma') d\tilde{\theta} d\tilde{\theta}' d^2\sigma d^2\sigma'} \tag{20}$$

[8]The IR divergence at $\kappa = 0$ is not what interests us here, and it is in fact unphysical. It must disappear in a more accurate treatment taking into account that t only varies within the finite range $[0, T]$.

Of course, the κ integral above is divergent for small *finite* values of ε, and the expression we've given must be understood in terms of analytic continuation in ε. As we'll see below, an analogous analytic continuation is needed for the annulus modular integration. This is very similar to what has been described in [14].

Now, in the path integral of F_2, the expression (20) will appear in convolution with $\partial_n \partial_{n'} D(\theta, \theta')$, and, since the latter is orthogonal to constant modes, the path integral of F_2 will not contain any UV-divergent piece. Both path integrals of F_1 and F_3 will simplify due to the relation

$$\int \partial_n D(\theta, \sigma) J^\mu(\sigma) d\theta d^2\sigma = -\int J^\mu d^2\sigma = p_1^\mu - p_2^\mu$$

and the UV divergent part of the path integral of F_3 will turn out to be of order $1/M^2$, not $1/M$ due to the relation $2\wp_1(\wp_1 - \wp_2) = (\wp_1 - \wp_2)^2$. If we also notice that the exponential in (20) is merely the value of the Gaussian integral

$$\int \mathcal{D}t(\theta)\, \exp\left[-S^{(2)}_{eff} - S^{(4)}_{eff}\right]$$

(without any insertions), we conclude that the UV divergent part of the disk amplitude to the order $1/M$ is proportional to

$$\frac{(p_1 - p_2)^2}{\alpha' M} A^0_{D_2} \sqrt{\log \varepsilon} \tag{21}$$

where $A^0_{D_2}$ is the zeroth order value of the disk amplitude, i.e. the amplitude computed in the background of a straight D-particle worldline, in neglect of recoil.

We now have to compare this result with the divergence in the modular integration of the annulus amplitude to the lowest order in $1/M$, i.e. the annulus amplitude in the background of a straight D-particle worldline. The computation of such an annulus amplitude is fairly standard. The fact that the disk amplitude $A^0_{D_2}$ makes an appearance in the expression (21) should be taken optimistically as far as the prospective cancellation of divergencies is concerned, since the familiar plumbing fixture construction of [22] relates the divergencies in the modular integration to amplitudes on worldsheets of lower genus. Let us see in some more detail how this actually works out.

Along the lines of [22], the annulus amplitude with an insertion of the operators $V^{(1)}, \cdots, V^{(n)}$ (in the interior) can be expressed through the disk amplitudes with additional operator insertions at the boundary as follows:

$$\left\langle V^{(1)} \cdots V^{(n)} \right\rangle_{\text{annulus}} = \sum_\alpha \int dq q^{h_\alpha - 2} \int d\theta d\theta' \left\langle V_\alpha(\theta) V_\alpha(\theta') V^{(1)} \cdots V^{(n)} \right\rangle_{D_2}$$

where the summation extends over a compete set of local operators $V_\alpha(\theta)$ with conformal weights h_α, and q is the gluing parameter that can be related to the annular modulus. The divergence in the integral over q coming from the region $q \approx 0$ (corresponding to an annulus degenerating into a ring) will be dominated by the terms with the smallest possible h_α. Neglecting the tachyon divergence, which is a pathology peculiar to the case of the bosonic string, we identify (in close relation to the investigations of [13]) the following set of relevant operators (parametrized by an integer $i \in [1, 26]$ and a real ω):

$$V^i(\theta, \omega) =: \partial_n X^i(\theta) e^{i\omega X^0(\theta)}:$$

with conformal weights $h = 1 + \alpha'\omega^2/4$. For small values of q (which is the region we're interested in) only small values of ω will contribute into the integral. We therefore transform the annular divergence as follows:

$$\begin{aligned}\left\langle V^{(1)}\cdots V^{(n)}\right\rangle_{\text{annulus}} &\sim \int dq d\omega q^{-1+\alpha'\omega^2/4}\int d\theta d\theta' \left\langle V^i(\theta,\omega)V^i(\theta',\omega)V^{(1)}\cdots V^{(n)}\right\rangle_{D_2} \\ &\sim \int dq d\omega q^{-1+\alpha'\omega^2/4}\int d\theta d\theta' \left\langle V^i(\theta,0)V^i(\theta',0)V^{(1)}\cdots V^{(n)}\right\rangle_{D_2} \\ &\sim (p_1-p_2)^2\left\langle V^{(1)}\cdots V^{(n)}\right\rangle_{D_2}\int dq\, d\omega q^{-1+\alpha'\omega^2/4}\end{aligned}$$

where we've taken into account that the operator $\int \partial_n X(\theta)d\theta$ merely shifts the (straight) D-particle worldline and inserting it into any amplitude amounts to multiplication by the total transferred momentum $p_1 - p_2$. If we now cut off the $dq\, d\omega$-integral at $q = \varepsilon$, the regularized value is proportional to $\sqrt{\log\varepsilon}$. Since q is introduced as a distance along the boundary of the worldsheet, the geometrical meaning of ε here is precisely the same as in the formula (21). Recalling that there is an additional power of g_{st} in the annulus amplitude as compared to the disk amplitude with the same insertions, we conclude that the modular integration divergence is proportional to

$$g_{st}\,(p_1-p_2)^2\left\langle V^{(1)}\cdots V^{(n)}\right\rangle_{D_2}\sqrt{\log\varepsilon} \equiv g_{st}\,(p_1-p_2)^2 A^0_{D_2}\sqrt{\log\varepsilon}$$

To the order $1/M$ (i.e. to the order g_{st}), we recognize precisely the same structure, as in (21). We shall not work out here the actual coefficients that depend on a number of conventions, but it is clear that the non-trivial dependencies on the momenta and the cut-off do in fact match, as it is necessary for the successful implementation of the Fischler-Susskind mechanism.

One cannot help but noticing that the version of the Fischler-Susskind mechanism that we've just described bears a strong resemblance to the investigations of [14]. Indeed, limiting themselves to the lowest order of the string coupling expansion, the authors of [14] have shown that, to that particular order, the consistency of the string S-matrix should be restored if the background of a straight D-particle worldline is augmented by an inclusion of the operator

$$V_{TF} = \frac{p_1^\mu - p_2^\mu}{M}\int \partial_n X_\mu(\theta)X^0(\theta)\,\Theta(X^0(\theta))d\theta$$

(where $\Theta(X^0)$ is a step function). Heuristically, such an operator corresponds to the D-particle abruptly starting to move with the appropriate recoil velocity at the moment $X^0 = 0$. (Once again, the operators $\int \partial_n X^i(\theta)d\theta$ with i being one of the Dirichlet directions shift the entire trajectory of the D-particle.) The UV divergence induced by this operator is precisely what we've found in (21). Moreover, the divergent integral

$$\int \frac{d\kappa}{\kappa^2}e^{-\kappa^2\log\varepsilon}$$

makes an actual appearance in the derivations of [14]. These parallels should not be too surprising since, for example, the structure

$$\int\limits_0^t d(\tilde{t},\sigma)J^\mu(\sigma)d\tilde{t}d^2\sigma$$

(entering the expression for the saddle point value of the D-particle trajectory $f^\mu(t)$) develops a discontinuity in its first derivative as the boundary of the worldsheet shrinks to a single point.

The authors of [14] have remarked that the abruptness in the change of the D-particle's velocity inherent to their scenario should probably be alleviated at the higher orders of the recoil perturbation theory. In fact our present formalism bypasses this problem from the very beginning. The summation over all possible D-particle trajectories automatically gives rise to the UV divergence required by the Fischler-Susskind mechanism, making it completely unnecessary to introduce the abrupt recoil by hand.

Is it possible to reach a deeper understanding of how the Fischler-Susskind mechanism works in our present formalism beyond the lowest order of the recoil perturbation theory? Extending our above analysis to the higher orders in g_{st} is likely to require a more systematic picture of both the general structure of the recoil perturbation theory and the general structure of the divergencies in the modular integration. Yet, conceptually, there doesn't seem to be anything puzzling about how the cancellations could actually occur.

Of course, the situation is much more intricate for the annihilation/pair-production case. In that regime, the first non-trivial UV divergence will arise if we try to compute the pre-exponential factor in the saddle point estimate (15). There doesn't appear to exist any meaningful expansion of this pre-exponential factor in any small parameter. It is merely a determinant of some integral operator, whose divergent part needs to be matched against the modular divergencies from the higher genus worldsheets. Any cancellations in this setting will necessarily be rather subtle. In particular, they may require certain relations between the values of the saddle point exponentials, and therefore certain relations between the solutions to the saddle point equation (11) for worldsheets of different genera. We shall not pursue this line of thought any further in this present investigation. Even though the considerations related to the Fischler-Susskind mechanism are of a crucial importance for establishing the consistency of the worldline formalism, they do not affect the estimates of the saddle point exponential and, in particular, its coupling constant dependence, which is the main result of this paper.

Emission of a large number of strings

One may be somewhat worried by the fact that the annihilation amplitudes computed via the saddle point method are suppressed much stronger in the limit $g_{st} \to 0$ than $\exp(-O(1/g_{st}))$, which is generally expected to be the order of the non-perturbative corrections in string theory [23]. The resolution of this apparent paradox is that the actual annihilation is most likely to proceed into a large ($\sim 1/g_{st}$) number of quanta, whereas the number of strings in the final state has been kept fixed as $g_{st} \to 0$ in our preceding considerations. Indeed, as we could see in the above, the annihilation process was dominated by worldsheets of size $\sim 1/g_{st}$, and it would hardly be natural for such extended configurations to decay into a small number of strings, a circumstance responsible for the anomalously strong suppression of the amplitude (15).

We shall now obtain some estimates of the amplitude for the D-particles to annihilate into a large number of quanta, and show that those contributions are in fact likely to saturate

the bound of [23]. The analysis of the previous sections still applies, but the position of the saddle point (and hence the functional $\mathcal{F}$) now depends on g_{st}, and therefore extra care needs to be taken in making statements about the coupling constant dependence of the amplitude.

It can be seen by inspecting the expression (15) that, unless all but an infinitesimal fraction of the final state strings are emitted from disconnected worldsheet components of disk topology with exactly one vertex operator on each of them, the g_{st}^{χ} factor itself will provide suppression of order $\exp(O(\ln(g_{st})/g_{st}))$. The extent to which the dynamics (accounted for by the saddle point estimates) enhances this suppression depends on the topology of each particular diagram. When most of the final state strings are emitted from disks with a single vertex operator, the non-perturbative suppression can only arise from the value of the saddle point exponential. We shall now estimate the contribution from those diagrams.

When the worldsheet consists of many disconnected components each of which carries away only a small fraction of the momentum, the independent displacements of those disconnected components cause only a small change of the saddle point function. We therefore encounter quasi-zero modes very similar to the ones described above (14). One way to account for these quasi-zero modes is to introduce the integration over them explicitly into the path integral by inserting $\delta(\int_i t(\theta)d\theta - \bar{t}_i \int_i d\theta)$ for each of the disconnected components of the worldsheet (the θ integral runs over the ith component of the worldsheet, $\bar{t}_i$ thereby being the average position of its boundary). The variational problem of (11-12) has then to be solved subject to the constraints specified by the δ-functions. (This would introduce the corresponding Lagrange multipliers into the right-hand side of (11), but they are inessential to our subsequent considerations.)

In order to obtain constraints on the value of the amplitude, let us expand the $\mathcal{P}(t)$ in (11) in Taylor series around $\bar{t}_i$ (for each i). It is easy to see that unless more than $O(1/g_{st}^{1-\varepsilon})$ (with ε arbitrarily small and positive) of the $\bar{t}_i$'s come into an interval of size $O(g_{st})$, the solution to (11) for θ belonging to the ith disconnected component is given by[9]

$$t(\theta) = \bar{t}_i + \frac{1}{4}\int \left[\partial_n \partial_{n'} D(\theta,\theta')\right]^{-1} \partial_{n'} D(\theta',\sigma')\mathcal{P}^{\mu}(\bar{t}_i)\mathcal{J}_{\mu}(\sigma')d^2\sigma' d\theta' + O(g_{st}^{1+\varepsilon})$$

(The second term here is closely related to the saddle point which dominates the recoil perturbation theory in the lowest order of g_{st}.) For the value of the saddle point function (the exponential in the last line of (10)), we obtain

$$-\pi\alpha' M^2 \left(\int_0^T dt\, z(t)\mathcal{P}^2(t) + \int d\theta d\theta' \partial_n \partial_{n'} D(\theta,\theta') \int_0^{t(\theta)} \mathcal{P}(t)dt \int_0^{t(\theta')} \mathcal{P}(t')dt' \right)$$

$$= -2\pi i\, \alpha' M^2 \sqrt{\wp^2}\, T$$

$$+\frac{\pi\alpha'}{4}\sum_i \int d\theta d\theta' d^2\sigma d^2\sigma' \mathcal{P}^{\mu}(\bar{t}_i) J_{\mu}(\sigma)\partial_n D(\theta,\sigma)\left[\partial_n\partial_{n'} D(\theta,\theta')\right]^{-1} \partial_{n'} D(\theta',\sigma')\mathcal{P}^{\nu}(\bar{t}_i) J_{\nu}(\sigma')$$

$$+O(1/g_{st}^{1-\varepsilon})$$

[9]We are essentially demanding the variation of $\mathcal{P}$ to be small over the extent of a single disconnected component of the worldsheet, and then use the expression for the saddle point value of $t(\theta)$ in the background of a straight worldline.

The second line here merely provides for the correct pole structure (and disappears after the application of the reduction formula). Each term in the sum from the third line is of order $O(g_{st}^0)$ and negative ($\mathcal{P}$ is imaginary to the lowest order in g_{st}). However, since there are $O(1/g_{st})$ terms, we should expect a non-perturbative suppression of order $\exp(-O(1/g_{st}))$.

We still have to deal with the excluded region of integration over the $\bar{t}_i$'s, namely, the configurations with more than $O(1/g_{st}^{1-\varepsilon})$ of the $\bar{t}_i$'s in an interval of size $O(g_{st})$. The volume of this region is proportional to $\exp(O(\ln(g_{st})/g_{st}^{1-\varepsilon}))$, however, and this is the amount of non-perturbative suppression provided regardlessly of the value of the saddle point exponential. Barring any unfathomable cancellations, the sum of the contributions from the two regions would be of order $\exp(-O(1/g_{st}))$, in accordance with the general estimates of the non-perturbative effects in string theory.

Bosonic D-particle decay

It is a notable feature of the worldline formalism that most of the results obtained in the previous sections are immediately generalized to the case of bosonic D-particle decay that has received a large amount of attention within the tachyon condensation considerations. Indeed, the natural proposal for the D-particle decay amplitude is

$$\langle k_1, \cdots, k_m | p\rangle = \lim_{p^2 \to -M^2} \int dx\, dx'\, e^{ipx}\, G(x, x' |\, k_1, \cdots, k_m) \tag{22}$$

where $G(x, x')$ is as it has been defined in (1). The meaning of this formula is that the endpoint of the D-particle worldline is unconstrained in the path integral, and the reduction formula is applied only to the starting point. The path integral thus describes a "disappearing" D-particle.

Given the formal similarities between the expressions (2) and (22), it is not surprising that the derivations presented above for the annihilation/pair-production case carry over to the decay process with a formal substitution $p_1 = p$, $p_2 = 0$. One important difference is that, for $p_1^2 \neq p_2^2$ the quasi-zero mode described in the paragraph above (14) will be absent. The saddle point configuration of the worldsheet will be localized near the endpoint of the D-particle worldline, as one would expect on general grounds. For that reason, the factor of T featured in (14) will not make an appearance for the case of D-particle decay. This is reassuring, since it precisely replaces the double pole necessary for the application of the reduction formula (2) by a single pole necessary for the application of the reduction formula (22). The conclusions regarding the coupling constant dependence of the amplitude to the leading order of the saddle point approximation will remain intact, i.e. the exclusive decay amplitudes will be non-perturbatively suppressed as $\exp[-O(1/g_{st}^2]$.

It would be interesting to see to which extent the results obtained here are compatible with the investigations of the closed string emission by a decaying D0-brane within the tachyon condensation approach [4]. Unfortunately, the (non-self-consistent) neglect of the closed string back reaction that underlies the derivations of [4] makes it hard to judge which features of the emission spectrum described there should be trusted.

Conclusions and speculations

As the calculations of the previous sections reveal, the problem of D0-brane annihilation appears in many ways much more tractable in our present context than the field-theoretical soliton-anti-soliton annihilation or, more generally, any other topological defect annihilation process that the author is aware of. Indeed, the saddle point method provides rigorous results regarding the coupling constant dependence for the annihilation with a fixed number of final state quanta, and certain qualitative estimates can be devised for final states whose multiplicity increases as g_{st} goes to zero. For a fixed number of final state closed strings (and for D0-brane pair production), the amplitudes turn out to be suppressed as $\exp[-O(1/g_{st}^2)]$, much stronger than the naïve expectation $\exp[-O(1/g_{st})]$.

For the case of closed string scattering off a D-particle, the worldline techniques developed here offer a rather elegant construction of the expansion in string coupling constant that properly takes into account the D-particle recoil. In particular, we resolve the paradox related to the abruptness in the change of the D-particle velocity that made an unwelcome appearance in the previous attempts to address the same problem [14].

It is a somewhat dissatisfying feature of our present formalism that only *exclusive* amplitudes appear to be computationally accessible. Should one have had actual D-particles at one's disposal, exploring the refined properties of their annihilation (such as final state multiplicity distribution) would certainly provide a valuable insight into their nature. Exclusive annihilation amplitudes would be *the* relevant theoretical prediction to guide this sort of experimentation. However, in most phenomenological applications (such as cosmology), the most important quantity is the simplest characteristic of the annihilation process, namely, the total cross-section.

It is the non-trivial nature of the final state multiplicity distribution, i.e. the dominant role of the final states with a huge number of quanta, that makes the reconstruction of the total cross-section from the exclusive annihilation amplitudes so hard. If one could overcome this difficulty, it would be possible, for example, to examine in our setting the stretched D-string interconnection, a process that has so far only admitted a semi-quantitative treatment [24]. The generalization of our present formalism to the case of D-strings is not likely to pose much difficulty. The problem is that, in the cosmological context, one is interested in the *total* interconnection probability.

For the case of D-particle pair production, we have established a *very strong* non-perturbative suppression (of order $\exp(-1/g_{st}^2)$). It would be interesting to contemplate whether it may have anything to do with the exponential suppression of the microscopic black hole pair production amplitudes argued by one of the sides in the dispute [8].

Another amusing feature is that there does not appear to be a unitarity relation that constrains the absolute normalization of the D-particle annihilation amplitude. It is hard to judge at this point whether or not this represents an actual non-perturbative ambiguity in string theory.

Since many qualitative features of the topological defect annihilation, such as the non-perturbative suppression of the exclusive amplitudes and the high mean multiplicity of the final state, appear to be rather general, it is natural to ask whether our computation can shed any light on the field-theoretical soliton annihilation, a process that has thus far defied an analytic treatment. The Christ and Lee formalism of [12] provides a rather conspicuous

link to our approach, as it introduces explicitly the center-of-mass position of the soliton, a worldline to be integrated over. It is certainly true that performing the path integral over the fluctuations of the fields around the given trajectory of the soliton is a much more difficult task than the worldsheet integrations pertinent to the D-particle case. This circumstance is also likely to undermine the direct applicability of the saddle point method, since it seems to hinge upon the summation over the worldlines. Yet, one could try to develop some intuition about the kinds of worldlines that contribute most, and establish some bounds, say, on the coupling constant dependence. If such a program succeeds, it would effectively provide a path-integral derivation of certain non-trivial properties of the classical annihilation solution. Mathematically and physically, such a derivation would present a fair amount of elegance.

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